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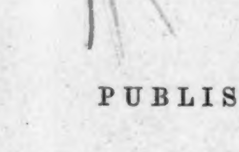
THE MATHEMATICAL MONTHLY.

FEBRUARY, 1859.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.



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THE

MATHEMATICAL MONTHLY.

Vol. I....FEBRUARY, 1859....No. V.

PRIZE PROBLEMS FOR STUDENTS.

I.

The abscissa and double ordinate of a segment of a common parabola are a and b, and the diameters of its circumscribed and inscribed circles D and d; to prove that $D+d=a+b$.

II.

A great circle of the sphere passes through two given points; find the rectangular coördinates of its pole.

III.

If the two sides of a movable right angle are always tangents to a given ellipse, its summit will describe a circle concentric with the ellipse, the radius of which is equal to the chord joining the extremities of the major and minor axes.

IV.

If a circle be described through the foci of an ellipse and any point in the conjugate axis produced; to prove that the right line joining that point and one of the points where the circle cuts the ellipse, will be a tangent to the ellipse.

V.

If D represent any diameter of an ellipse, and P the parameter of D, to find when $D + P$ is the least, and when the greatest, possible.

The solution of these problems must be received by the first of April, 1859.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS, IN NO. I. VOL. I.

The first Prize is awarded to G. W. Hill, of the Senior Class of Rutgers' College, New Brunswick, N. J.

The second Prize is awarded to Waller Holladay, Student of Mathematics, in the University of Virginia.

Prize Solution of Problem I.

" Find θ from each of the equations

(1) $$\tan\theta \tan 2\theta + \cot\theta = -2,$$

(2) $$2\sin^2 3\theta + \sin^2 6\theta = 2,$$

(3) $$\cos n\theta + \cos(n-1)\theta = \cos\theta."$$

Equation (1) multiplied by $\tan\theta$ becomes

(4) $$\tan^2\theta \tan 2\theta + 1 = -2\tan\theta\,;$$

and since $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta},$$

whence $$2\tan\theta = \tan 2\theta - \tan^2\theta\tan 2\theta,$$

we obtain, by substituting this value of $2\tan\theta$ in (4), and reducing

$$\tan 2\theta = -1$$

$$\therefore 2\theta = \tan^{-1}(-1) = n\pi + 135° = n\pi + \tfrac{3}{4}\pi \quad \therefore \theta = \frac{(4n+3)\pi}{8}.$$

All the roots are obtained by giving n all positive integer values.

Equation (2), since

$$2\sin^2 3\theta = 1 - \cos 6\theta, \text{ and } \sin^2 6\theta = 1 - \cos^2 6\theta,$$

becomes $$\cos^2 6\theta + \cos 6\theta = 0,$$
from which we get $\cos 6\theta = 0$, and $\cos 6\theta = -1$.

$$\therefore\ 6\theta = \cos^{-1} 0 = n\pi + 90^\circ = n\pi + \tfrac{1}{2}\pi \ \therefore\ \theta = \frac{(2n+1)\pi}{12}$$

$$\therefore\ 6\theta = \cos^{-1}(-1) = 2n\pi + 180^\circ = 2n\pi + \pi \ \therefore\ \theta = \frac{(2n+1)\pi}{6}.$$

Equation (3), since

$$\cos n\theta = \cos(n-1)\theta\cos\theta - \sin(n-1)\theta\sin\theta$$

$$\cos(n-2)\theta = \cos(n-1)\theta\cos\theta + \sin(n-1)\theta\sin\theta,$$

becomes by adding

$$\cos n\theta + \cos(n-1)\theta = 2\cos(n-1)\theta\cos\theta = \cos\theta;$$

therefore $\cos\theta = 0$, or $\cos(n-1)\theta = \frac{1}{2}$.

$$\therefore\ \theta = \cos^{-1} 0 = n\pi + 90^\circ = \frac{(2n+1)\pi}{2},$$

$$\therefore\ (n-1)\theta = \cos^{-1}\tfrac{1}{2} = 2m\pi \pm 60^\circ = 2m\pi \pm \tfrac{1}{3}\pi \ \therefore\ \theta = \frac{(6m \pm 1)\pi}{3(n-1)}.$$

These solutions are by Mr. O. B. Wheeler, student in the University of Michigan.

Prize Solution of Problem II.

"The whole surface of a right cone is three times the area of the base. Find the vertical angle."

The convex surface is twice the area of the base. But the convex surface = perimeter of base × ½ slant height, and the area of base = perimeter × ½ radius. Therefore, since one area is twice the other, slant side = twice radius of base. ∴ the section containing the axis is an equilateral triangle, and the vertical angle is 60°.

This solution was given by Messrs. Everett, Hill, Palfrey, and Tower.

Prize Solution of Problem III.

"The sum of the squares of the reciprocals of two radii vectores from the centre of an ellipse at right angles to each other is constant; the perpendicular from the centre, on the chord joining their extremities, is also constant. What part of the area of the ellipse is the circle whose radius is this perpendicular?"

The equation of the ellipse, referred to its centre and axes, is $A^2 y^2 + B^2 x^2 = A^2 B^2$, which, for $x = r \cos \varphi$ and $y = r \sin \varphi$, becomes its polar equation,

$$r^2 = \frac{A^2 B^2}{A^2 \sin^2 \varphi + B^2 \cos^2 \varphi};$$

the centre being the pole, and the transverse axis the prime radius. For r', the radius vector perpendicular to r, φ must be increased by $\frac{1}{2}\pi$.

$$\therefore\ r'^2 = \frac{A^2 B^2}{A^2 \sin^2 (\varphi + \frac{1}{2}\pi) + B^2 \cos^2 (\varphi + \frac{1}{2}\pi)} = \frac{A^2 B^2}{A^2 \cos^2 \varphi + B^2 \sin^2 \varphi},$$

$$\therefore\ \frac{1}{r^2} + \frac{1}{r'^2} = \frac{A^2 + B^2}{A^2 B^2} = \frac{r'^2 + r^2}{r'^2 r^2} = \text{a constant.}$$

Let p denote the perpendicular; $\sqrt{r^2 + r'^2}$ is the chord; therefore

$$p \sqrt{r^2 + r'^2} = r r',$$

since both expressions denote the double area of the same triangle.

$$\therefore\ p^2 = \frac{r^2 r'^2}{r^2 + r'^2} = \frac{A^2 B^2}{A^2 + B^2} = \text{a constant.}$$

But $A B \pi =$ area of ellipse, and $\frac{A^2 B^2 \pi}{A^2 + B^2} =$ area of circle.

$$\therefore\ \frac{\text{area of circle}}{\text{area of ellipse}} = \frac{A B}{A^2 + B^2} = \text{the required part.}$$

If from the extremity B of the conjugate axis lines be drawn to A and A' the extremities of the transverse axis, and the angle $A B A' = \theta$, then

$$\frac{A}{\sqrt{A^2 + B^2}} = \sin \tfrac{1}{2}\theta, \text{ and } \frac{B}{\sqrt{A^2 + B^2}} = \cos \tfrac{1}{2}\theta.$$

$$\therefore\ \frac{A B}{A^2 + B^2} = \sin \tfrac{1}{2}\theta \cos \tfrac{1}{2}\theta = \tfrac{1}{2} \sin \theta.$$

$$\therefore\ \text{area of circle} = \tfrac{1}{2} \sin \theta \times \text{area of ellipse.}$$

This solution combines those given by Messrs. EVANS and EVERETT; the former gave the method of finding p^2, and the latter showed that the ratio of the areas equals $\frac{1}{2} \sin \theta$.

Prize Solution of Problem IV.

"Two circles, whose radii are R and r, touch each other externally. If θ is the angle included between the common tangents to the two circles, prove that $\sin\theta = \frac{4(R-r)\sqrt{Rr}}{(R+r)^2}$."

The construction readily shows, that in the right triangle of which the hypothenuse and a side are $R+r$ and $R-r$, that the angle opposite $R-r$ is $\frac{1}{2}\theta$. Therefore

$$\sin\tfrac{1}{2}\theta = \frac{R-r}{R+r}, \text{ and } \cos\tfrac{1}{2}\theta = \sqrt{1-\sin^2\theta} = \frac{2\sqrt{Rr}}{R+r},$$

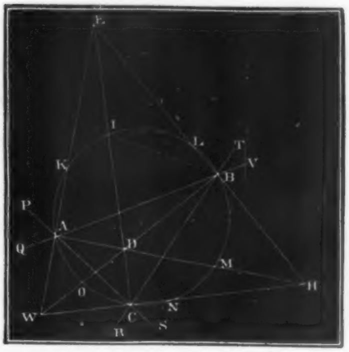
$$\therefore \sin\theta = 2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta = \frac{4(R-r)\sqrt{Rr}}{(R+r)^2}.$$

This is substantially the solution given by all the competitors.

Prize Solution of Problem V.

"Four circles may be described, each of which shall touch the three sides of a triangle, or those sides produced. If six straight lines be drawn, joining the centres of these circles two and two, prove that the middle points of these six lines are in the circumference of the circle circumscribing the given triangle."

Let ABC be the given triangle. Bisect its angles by the lines AH, BW, CE. These lines contain the centres of the tangent circles, since each line bisects the angle formed by two tangents. D is the centre of the inscribed circle, and the centres of the three other circles, H, W, and E are determined by drawing HW, WE, and EH perpendicular respectively to CE, AH, and BW; for by this construction HW, WE, and EH are made to bisect, respectively, the angles BCS, CAQ, and ABT formed by tangents to the circles, and therefore contain the centres of the circles. Circumscribe about the triangle the circle $AIBNO$, and draw IB.



The angle IBO is measured by $\frac{1}{2}(IA + AO)$; also the angle IDB is measured by $\frac{1}{2}(IB + OC)$. But $IB = IA$ and $OC = AO$.

$\therefore$ the angle $IBO = IDB$ and their complements $EBI = IEB$.

$\therefore$ the triangles BDI and EIB are isosceles, and $EI = IB = ID$.

Similarly it may be proved that $MH = MD$ and $WO = OD$. Again, from the secants AE and EC, $\frac{EI}{EK} = \frac{EA}{EC}$, and from the similar triangles EAD and ECW, $\frac{EA}{ED} = \frac{EC}{EW}$. Combining these two proportions, $\frac{EI}{ED} = \frac{EK}{EW}$ or $\frac{EI}{ED - EI} = \frac{EK}{EW - EK} \therefore \frac{EI}{ID} = \frac{EK}{KW}$; but $EI = ID \therefore EK = KW$.

Similarly it may be proved that $WN = NH$ and $HL = LE$. Hence the middle points of the lines connecting the centres of the four tangent circles are in the circumference of the circumscribing circle.

This solution is by Mr. George A. Osborne, Jr. Several other solutions of this interesting problem are also of decided excellence; and it is only for want of room in the Monthly that we do not recommend them for publication. The analytical solutions of Messrs. George B. Hicks and George W. Jones, although long, are of a high order of merit.

No complete sets of solutions of the Prize Problems in the second number of the Monthly have been received; and none of the competitors are entitled to a prize.

Joseph Winlock,
Chauncey Wright,
Truman Henry Safford.

NOTE ON THE PROPOSITION OF PYTHAGORAS.

By Rev. A. D. Wheeler, Brunswick, Maine.

The truth of this proposition may be shown mechanically, by means of very simple apparatus. Two methods are here presented.

1. Cut from wood or pasteboard four equal right-angled triangles; and three squares, corresponding to the three sides of one of those triangles. They may be disposed as in the figures below, and these figures are manifestly equal. Hence the larger square must be equal to the sum of the other two.

$$a^2 + b^2 + 2\,ab = c^2 + 2\,ab,$$
$$\therefore\ a^2 + b^2 = c^2.$$

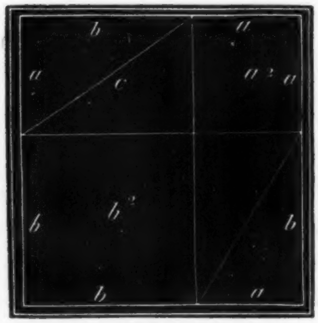


Fig. 1.

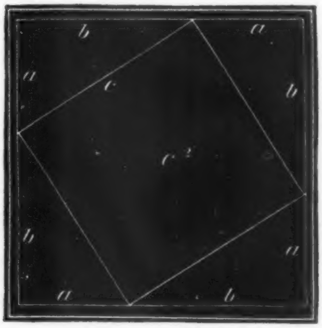


Fig 2.

2. From any suitable material, cut out two squares, joined at one of their sides, as in Fig. 3. From these squares cut off the triangles $A\,B\,C$, $B\,D\,E$, as in Fig. 4, making $A\,B = b$; whence there will remain $B\,D = a$. We have then the squares upon the base and perpendicular. Let the triangle $A\,B\,C$ turn on a hinge at C, until it comes into the position $C\,H\,F$, and the triangle $B\,D\,E$ turn on a hinge at E until it comes into the position $E\,G\,F$. We

have then the square on the hypothenuse, which must of course be equal to the sum of the other two squares, since it is constructed out of them.

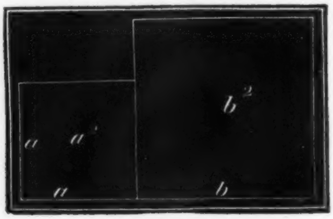


Fig. 3.

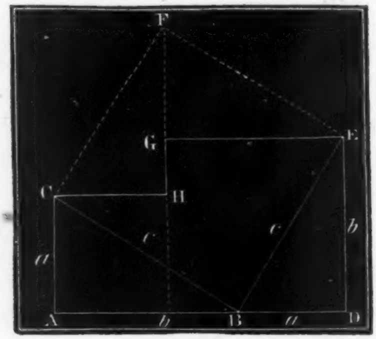


Fig. 4.

NOTE ON THE INTERPRETATION OF ALGEBRAIC RESULTS.

BY W. H. PARKER,
Professor of Mathematics in Middlebury College, Vermont.

In the discussion of algebraic problems, we sometimes find the result *zero*, arising from particular suppositions made upon the quantities that enter into the value of x.

What is the proper interpretation of this result? At the risk of seeming presumptuous in discussing a question which already has the seal of authority upon it, I will attempt to answer it in part, by considering two cases which arise in discussing the following familiar problem.

Problem. Upon a line on which two lights are placed, whose intensities at the distance 1, and whose distance apart, are given; it is required to find the point which is equally illuminated by them, assuming that the intensity of the same light, at different distances, varies inversely as the squares of the distances.

A B

Let $A\,B$, the distance between the lights, be represented by c;

and the distance of the required point from A considered as the origin of distances, by x. Let $a =$ the intensity of the light A at the distance 1; and $b =$ the intensity of B at the same distance. Then $\frac{a}{x^2} =$ the intensity of A at the required point; and $\frac{b}{(c-x)^2} =$ the intensity of B at the same point. Since the intensities are equal at that point, we have the equation $\frac{a}{x^2} = \frac{b}{(c-x)^2}$; whence $x = \frac{c\sqrt{a}}{\sqrt{a} \pm \sqrt{b}}$.

If we suppose $c = 0$, and $a > b$, or $a < b$, both values of x reduce to 0. How shall we interpret this? We are told, it shows that the points of equal illumination coincide with the one where the lights are placed. If $a > 2b$, obviously the interpretation would not be different.

This conclusion appears to me unsound. Let us examine it. If the lights are placed at A, and AS be a unit of distance,

A ¼ ½ S

then at ½ AS the intensities are $\left(\frac{a}{(\frac{1}{2})^2}\ \frac{b}{(\frac{1}{2})^2} =\right)$ $4a$ and $4b$; at ¼ the distance AS the intensities are $16a$, $16b$; at $\frac{1}{10}$ the distance $100a$, $100b$, and so on till we reach the limit which we are approaching, that is, the point where the lights are placed. By inspecting the expressions for the intensities, as we approach the point A, $4a$, $4b$; $16a$, $16b$; $100a$, $100b$, ... we see that their relative intensity is the same throughout. If $a = 100b$ at the distance AS, it $= 100b$ at the distance 0; which shows that when the lights are unequal, and occupy the same point on the line, that point cannot be as much illuminated by one light as by the other.

The unsoundness of the conclusion may be exhibited in another way. Let it be required in the problem to find the point which is *twice* as much illuminated by one light as by the other. The values of x now become $\frac{c\sqrt{a}}{\sqrt{a} \pm \sqrt{2b}}$. If we suppose $c = 0$, and $a > 2b$, or

$a < 2b$, both values of $x = 0$; which shows (following the interpretation adopted in the previous case) that the points, which are twice as much illuminated by one light as by the other, coincide with the point where the lights are placed.

In the former problem, when we were seeking the point of *equal* illumination, we found, even when $a > 2b$, that the point where the lights are placed is equally illuminated by them; now we find, when $a > 2b$, that the point where the lights are placed is twice as much illuminated by one as by the other. That is, the point is at the same time both equally illuminated by the two, and twice as much illuminated by one as by the other. A process which leads to these contradictory results must of course be at fault.

Shall we conclude, therefore, that algebraic reasoning is not always to be relied upon? By no means. Since all the transformations in algebraic equations are made by the use of axioms, no logical process is more simple or more reliable. If our primitive equation be true, all the resulting equations are necessarily true.

*The error to which I have invited attention is one of *interpretation.* We call to our aid the algebraic process, to determine the distance of the required point from the fixed point A. Algebra did not assume to prove that there is such a point (that was taken for granted in the very enunciation of the problem); but, if there be such a point, to determine its distance from the origin A. Now the result 0 shows that there is no such point on either side of the lights, and *shows nothing more.* Its language is "no distance." It affirms nothing. It merely denies; and denies only in respect to every other point on the line. To the question, is that point equally illuminated? it gives no answer. As in the discussion of a general problem, each new supposition converts it into a new and particular problem; it may happen that some of these will contain impossible conditions. The one we have been considering is of

* See p. 178.

this character. This we have already proved by the application of our assumed physical law, which shows, that, when the lights are together, if their intensities are unequal at any point on the line, they are unequal at every point.

The other case proposed is when $c = 0$, and $a = b$. Here the first value of $x = 0$. This is said to show that the point occupied by the lights is equally illuminated by them. But it seems to me simply to deny that the point of equal illumination between the lights is anywhere else; without affirming the existence of such a point. "No distance" is its language here, as in the former case. When by an inspection of the problem, or by a formal application of our physical law, we find the new problem possible, then the algebraic result shows the position of the point.

If, then, *zero* affirms nothing in regard to the possibility or impossibility of the conditions of the question, and (like infinity) is sometimes the answer to an impossible problem; when we come to interpret such a result, we are not to proceed upon the assumption that the thing required in the problem is possible, as we do when the result is a real and finite quantity; but are first to determine, by considering the nature of the question, whether the conditions are possible or not, and interpret the algebraic result accordingly.

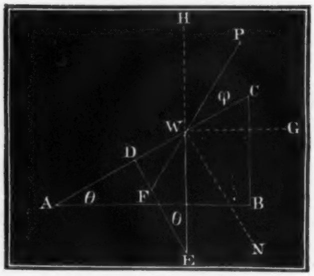
THEORY OF THE INCLINED PLANE, FOR ELEMENTARY INSTRUCTION.

BY THOMAS SHERWIN,
Principal of the English High School, Boston, Mass.

LET a heavy body, which we call W, of inappreciably small magnitude, be placed upon the inclined plane AC at the point W, and suppose that it is kept in equilibrium by a power P acting in the direction WP, AB being the horizontal base, and BC the

height of the plane. We are to investigate the relations of the power, weight, and pressure upon the plane.

Let θ be the angle of elevation of the plane, and φ the angle which the direction of the power makes with the plane. The weight acting vertically, through W draw an indefinite vertical line, and take on it WE equal to as many linear units as the weight contains units of weight. Through E draw ED perpendicular to AC, and produce PW until it meets ED in F.



The weight, represented in quantity and direction by WE, may, by the principles of the resolution of force, be resolved into two other forces, represented in quantity and direction by WF and FE, the two other sides of the triangle WFE. The force WF, acting directly opposite to the power, must, in case of equilibrium, be equal to the power, and the force FE, acting perpendicularly to the plane, produces pressure which is resisted by the plane. Call this pressure p. The power, weight, and pressure are then represented respectively by WF, WE, and FE.

Hence, $P:W = WF:WE = \sin FEW : \sin WFE$.

But the triangles ABC and DWE are similar, since each has a right angle, and since WE is parallel to BC. Therefore the angle $FEW = \theta$; and $\sin WFE = \sin WFD = \cos FWD = \cos \varphi$. Therefore,

(1) $$P:W = \sin\theta : \cos\varphi.$$

Again, $P:p = WF:FE = \sin\theta : \sin FWE$. But $FWE = DWE - FWD = 90° - \theta - \varphi = 90° - (\theta + \varphi)$; $\therefore \sin FWE = \sin[90° - (\theta + \varphi)] = \cos(\theta + \varphi)$. Hence,

(2) $$P:p = \sin\theta : \cos(\theta + \varphi).$$

Since the antecedents are alike in (1) and (2), the cosequents are proportional; $\therefore$

(3) $$W : p = \cos \varphi : \cos (\theta + \varphi).$$

The three proportions given above are general; that is, they are applicable, whatever angle the direction of the power may make with the plane.

Suppose, now, that the power acts parallel to the plane. In this case φ is zero, and (1), (2), and (3) become

(4) $$P : W = \sin \theta : \cos 0 = \sin \theta : R = BC : AC;$$

(5) $$P : p = \sin \theta : \cos \theta = BC : AB;$$

(6) $$W : p = \cos 0 : \cos \theta = R : \cos \theta = AC : AB.$$

Hence, when the power acts parallel to the plane, *the power is to the weight as the sign of elevation is to radius, or as the height of the plane is to the length; the power is to the pressure as the sine of elevation is to its cosine, or as the height of the plane is to its base; the weight is to the pressure as radius is to the cosine of elevation, or as the length of the plane is to its base.*

If the power acts parallel to the base, φ becomes equal in value to θ; but it is negative, since it is reckoned below the line WC. Hence, $\varphi = -\theta$, and (1), (2), and (3) become

(7) $$P : W = \sin \theta : \cos (-\theta) = \sin \theta : \cos \theta = BC : AB;$$

(8) $$P : p = \sin \theta : R = BC : AC;$$

(9) $$W : p = \cos \theta : R = AB : AC.$$

Hence, when the power acts parallel to the base, *the power is to the weight as the sine of the elevation is to its cosine, or as the height of the plane is to the base; the power is to the pressure as the sign of elevation is to radius, or as the height of the plane is to the length; the weight is to the pressure as the cosine of the elevation is to radius, or as the base of the plane is to the length.*

Observe that, whether the power acts parallel to the plane or parallel to the base, the power corresponds to the height, and the weight corresponds to that part of the plane parallel to which the power acts.

Suppose φ equal to the complement of θ; then (1) becomes $P:W = \sin\theta:\sin\theta$; and the power and weight are equal, as they evidently should be, since the power then acts vertically upwards. Likewise (2) becomes

$$P:p = \sin\theta:0 \therefore p = \frac{P\times 0}{\sin\theta} = 0.$$

If the power acts downward and perpendicularly to AC, $\varphi = -90°$, and (1) becomes

$$P:W = \sin\theta:0 \therefore P = \frac{W\times\sin\theta}{0} = \text{infinity};$$ and (3) gives

$$W:p = 0:\sin\theta \therefore p = \frac{W\times\sin\theta}{0} = \text{infinity}.$$

Thus it appears that the power, when it acts perpendicularly to the plane, is infinite, and that, if such a power were applied, the pressure would also be infinite. But the expression for the power is, in this case, to be regarded rather as a symbol of impossibility, for no two forces acting at right angles to each other can be in equilibrium.

The limits of possibility, consistent with equilibrium, for the direction of the power, are vertically upwards, and at right angles to the plane downwards. The direction may reach the former limit, and approach indefinitely near to the latter. The angle which these limiting directions make with each other is evidently the supplement of the elevation. Thus, $HWN = 180° - \theta$.

By examining proportions (1), (4), and (7), we see, since no cosine except that of zero can be so great as radius, that the ratio of the power to the weight is least when the power acts parallel to the plane. The power, therefore, acts most advantageously in this direction. This truth is manifest independently of analysis, since the power acts directly opposite to that part of the weight which tends to move the body down the plane.

If in (4) we call θ zero, we have

$$P:W = 0:R \therefore P = \frac{W\times 0}{R} = 0,$$

and the same supposition in (6) gives

$$W : p = R : R;$$

showing that, in this case, the power is zero, and that the pressure is equal to the weight, as they manifestly should be, since the plane is horizontal.

If in the same proportions we suppose $\theta = 90°$, we have

$$P : W = R : R, \text{ and } W : p = R : 0 \;\therefore p = \frac{W \times 0}{R} = 0.$$

Hence, in this case, the power and weight are equal, and the pressure is zero.

It follows also from (4) and (6), that, since the sine increases and the cosine decreases with the increase of the angle up to 90°, the ratio of the power to the weight is less, and the ratio of the pressure to the weight greater, the less the elevation of the plane. For the sake of simplicity we have supposed the weight to be of infinitesimal magnitude. But all that has been demonstrated is applicable to a body of any magnitude, provided the weight acts at the point W, and the power acts in a direction which would pass through the centre of gravity of the body.

NOTE ON TWO NEW SYMBOLS.

BY BENJAMIN PEIRCE,
Professor of Mathematics in Harvard College, Cambridge, Mass.

THE symbols which are now used to denote the Neperian base and the ratio of the circumference of a circle to its diameter are, for many reasons, inconvenient; and the close relation between these two quantities ought to be indicated in their notation. I would propose the following characters, which I have used with success in my lectures: —

 to denote ratio of circumference to diameter,

 to denote Neperian base.

It will be seen that the former symbol is a modification of the letter *c* (*circumference*), and the latter of *b* (*base*).

The connection of these quantities is shown by the equation,

$$\text{ᘏ}^{\text{ᘓ}} = (-1)^{-\sqrt{-1}}.$$

THE NOTATION OF ANGLES.

By James Mills Peirce, Cambridge, Mass.

Professor Peirce, in his work on Analytic Mechanics, has introduced a method of denoting angles by writing the letters which represent the sides, one above the other. Thus, the angle between the axes of x and y in a rectilinear coördinate-system is denoted by the symbol $\frac{y}{x}$, which may be read x-y.

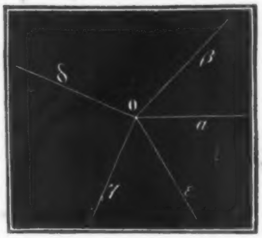
This method seems to answer, in the fullest possible manner, the purposes of a notation. It is at once simple and expressive. Each symbol, being determined by a principle and not chosen arbitrarily, carries its meaning on its face; and it is of course desirable that this should be true, as far as possible, of all notation, so that, in using general formulæ, we may not be under the necessity of looking up the significations of the symbols which they involve.

This system of notation may be somewhat further developed, and it will then be found to have other advantages besides those which have been pointed out.

1. The system may be made universal in its application by using *Greek letters* to denote the *directions* of lines, without reference to their length. Thus, if ρ denotes the axis in a system of polar coördinates, the polar angle will be $\frac{r}{\rho}$.

2. This notation affords a distinction between the two opposite circular directions which may be supposed to belong to the same angle. If a line be supposed to revolve about the point O, in the

accompanying figure, from the position α to the position β, its amount of rotation will be measured by the angle $\frac{\beta}{\alpha}$; but if it revolve from β to α, its rotation will be measured by $\frac{\alpha}{\beta}$. Since these rotations are equal in amount, but opposite in direction,



$$\frac{\beta}{\alpha} = -\frac{\alpha}{\beta};$$

that is, inverse angular symbols are negatives of each other.

3. By means of this notation, angles may be added by a mere inspection of the forms of their symbols. Thus we may write

$$\frac{\varepsilon}{\delta} + \frac{\gamma}{\varepsilon} + \frac{\alpha}{\gamma} + \frac{\beta}{\alpha} = \frac{\beta}{\delta};$$

for this is only saying that if a line rotate from δ to ε, then from ε to γ, thence to α, and thence to β, the resultant rotation is measured by the angle $\frac{\beta}{\delta}$. Again, by § 2,

$$\frac{\gamma}{\varepsilon} - \frac{\gamma}{\alpha} - \frac{\delta}{\varepsilon} + \frac{\beta}{\alpha} = \frac{\varepsilon}{\delta} + \frac{\gamma}{\varepsilon} + \frac{\alpha}{\gamma} + \frac{\beta}{\alpha} = \frac{\beta}{\delta}.$$

Hence, if a polynomial which consists of angular symbols can be so arranged, that, when all its terms are made positive, the upper letter of the first term is the lower letter of the second, the upper of the second the lower of the third, &c., the polynomial is equivalent to the angle made by the upper line of the last term with the lower line of the first. The same principle may be used to *decompose* angles.

This proposition is identical with Hamilton's Theorem of Versions. (Quaternions, Arts. 49, 65, &c.)

4. It is no objection to the rule of § 3, that it leaves a doubt as to whether the angles are to be measured in the most natural manner; that is, so as to be less than 180°. This ambiguity does not arise from the notation, but is inherent in the very notion of an angle, which may always have any one of an infinite series of values, differing by 360°. When, however, an angle is

treated through its trigonometric functions, this ambiguity may be disregarded.

5. Some of the above remarks may be illustrated by a solution of the problem, *To transform from one system of rectangular coördinates in a plane to another.*

Let the old system be that of x, y; let the new system be that of x_1, y_1; and let the coördinates of the new origin, referred to the old system, be x° and y°.

Then, by § 3, $\quad \frac{y_1}{x} = \frac{x}{x} + \frac{y_1}{x_1} = \frac{1}{2}\pi^* + \frac{x_1}{x}.$

The projections of x_1 and y_1 on the axis of x are respectively

$$x_1 \cos \tfrac{x_1}{x}, \quad y_1 \cos\left(\tfrac{1}{2}\pi + \tfrac{x_1}{x}\right) = -y_1 \sin \tfrac{x_1}{x};$$

and their projections on the axis of y are respectively

$$x_1 \sin \tfrac{x_1}{x}, \quad y_1 \sin\left(\tfrac{1}{2}\pi + \tfrac{x_1}{x}\right) = y_1 \cos \tfrac{x_1}{x}.$$

Hence the projections of the broken line formed by x_1 and y_1 on the axes of x and y are respectively

$$x - x^\circ = x_1 \cos \tfrac{x_1}{x} - y_1 \sin \tfrac{x_1}{x},$$
$$y - y^\circ = x_1 \sin \tfrac{x_1}{x} + y_1 \cos \tfrac{x_1}{x};$$

and we have

$$x = x^\circ + x_1 \cos \tfrac{x_1}{x} - y_1 \sin \tfrac{x_1}{x},$$
$$y = y^\circ + x_1 \sin \tfrac{x_1}{x} + y_1 \cos \tfrac{x_1}{x}.$$

6. The proposed symbol gives no more difficulty in printing than an ordinary fraction. For instances in which it occurs elevated or depressed out of the line, see PEIRCE's Analytic Mechanics, pp. $52_{13, 31}$, $53_{2, 5, 8}$, 101_3, &c., &c.

This notation is recommended to the attention of mathematicians.

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* See Prof. PEIRCE's Note on page 167. Of the desirableness of especial symbols to denote the quantities named, there can be but little doubt; and those suggested possess one essential requisite of a good notation, facility of use, as Prof. PEIRCE's experience in the lecture room proves. Besides, as obvious modifications of *c* and *b*, they can be easily distinguished and remembered. We hope to see them exclusively adopted in the Monthly. For the advantages of this "Notation of Angles," see the valuable work on Analytic Geometry by the Author of this paper. — ED.

MATHEMATICAL PRINCIPLES OF DIALING.

BY GEORGE EASTWOOD,
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My object in preparing these papers on Dialing is simply to bring into the small space of a few pages what the student might otherwise be obliged to seek through many volumes. This being my sole aim, and claiming nothing for these papers on the score of originality, I have not thought it necessary to give credit even in those cases where there is no doubt about the authorship.

I. HORIZONTAL DIALS.

(1) It is not proposed in this place to enter into a history of Sun-dials. The invention of more exact and more accurate methods of measuring time, for all practical and scientific purposes, has, in a great measure, superseded their use, and deprived them of much of that interest and importance which were once ascribed to them. But, although their utility has been superseded by clocks and watches, the mathematical principles upon which they were and may be constructed remain unimpaired, and are eminently calculated to amuse and instruct the aspiring student.

(2) The following definitions ought not to be lost sight of:—

A *horizontal dial* is one that is traced on a horizontal plane.

A *vertical dial* is one that is constructed on a vertical plane. It may be *east*, *west*, *north*, or *south*, according to the cardinal point which it may face.

Vertical declining dials do not face any one cardinal point.

Oblique dials are those constructed on planes which make oblique angles with the horizon. They have the name of *reclining dials* when they lean backwards from the observer, and *proclining* when they project forward.

An *equinoctial dial* is that whose plane is perpendicular to the earth's axis, or parallel to the equator.

The *declination* of a plane is an arc of the horizon comprised between the plane and the plane of the prime vertical.

The *azimuth* of a plane is the arc of the horizon comprised between the plane and the plane of the meridian, and is the complement of the declination.

The *meridian* of a plane is *that* meridian plane which is perpendicular to the plane of the dial. This plane differs from the meridian of the place, the latter being always perpendicular to the horizon.

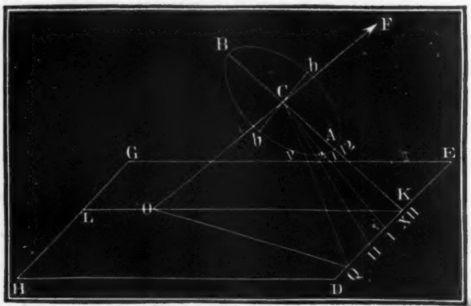
The *substyle* of a dial is the common section of its plane and the plane of its meridian, or it is the projection of the *style* of the dial upon its plane. In horizontal and in vertical south and north dials, the substyle coincides with the twelve o'clock hour line, but not in declining dials.

The *difference of longitude* of a dial plane is the angle which the plane of its meridian makes with the meridian of the place.

The *latitude* of a dial plane is the angle which the axis makes with the plane; this is the latitude of the place where the dial would be a horizontal one.

The *style* or *axis* of a dial must always point to the pole of the heavens.

(3) The equinoctial dial being the simplest of all dials to construct, and the horizontal dial ranking next to it, we will begin our investigations with the latter.



In the annexed diagram, then, let $G\,E\,D\,H$ be a horizontal plane, on which a dial is to be traced; $L\,O\,K$ a meridian line, $O\,C\,F$ a straight line or rod in the plane of the meridian, pointing to the pole, and making with $O\,K$ the angle $F\,O\,K$, equal to the latitude of the dial. Suppose $B\,A\,P$

to be an equinoctial dial, OCF its axis, and C its centre. Produce CA, the meridian line on this dial, to meet LK in K. For obvious reasons, the plane of the shadow will turn uniformly about the axis OCF, meeting the equinoctial plane in some line CPQ, and the horizontal plane in an analogous line OQ. Let C_1, C_2, &c., be the hour lines after noon on the equinoctial dial, and OI, OII, &c., the corresponding lines on the horizontal dial; the former will make, with the meridian line CAK, angles proportional to the time from noon, and will be known when the hour is given, 15° being counted to the hour. Suppose now the plane of the equinoctial dial to be extended till it meets the horizontal plane in the line QK; this line of intersection of the two planes is obviously perpendicular to the meridian lines CK, OK. The problem to be resolved is, therefore, to find the hour angle KOQ at the centre of the horizontal dial, corresponding to any given angle KCQ at the centre of the equinoctial dial, which measures the time from noon.

Let $h =$ hour angle KCQ on the equinoctial dial,

$h' =$ hour angle KOQ on the horizontal dial;

then the right triangles CKQ, OKQ give

$$KQ = CK \tan h,$$
$$KQ = OK \tan h'.$$

Put angle $KOC = \beta =$ latitude of the place for which the dial is to be constructed; then, C being a right angle, we have

$$CK = OK \sin \beta,$$
$$\therefore \tan h' = \tan h \sin \beta,$$

is the general equation of the hour angles on a horizontal dial, for any latitude.

(4) If the dial, instead of being horizontal, be required to be a vertical north or south dial, a very slight consideration will convince the young student, that, if the vertical south dial were carried to a

place whose latitude is the complement of the given latitude, it would be a horizontal one for that place. The equation of the hour angles on a vertical south or north dial, for latitude β, is, therefore,

$$\tan h' = \tan h \cos \beta .$$

(5) Vertical east and west dials are described on vertical planes which coincide with the meridian plane. The style or axis of a vertical east or west dial is parallel to its plane at any given height above it. As the plane of its shadow turns uniformly about its axis, it will cut off from the line, which is perpendicular to the six o'clock hour line, distances which will be the tangents of the angles generated by the shadow; that is, the tangents of the hour angles from six. If therefore $d =$ height of the style, $h =$ the hour angle from six, and $h' =$ distance cut off by the shadow, then

$$h' = d \tan h.$$

(6) And if h be taken for the hour angle from the twelve o'clock hour line, then

$$h' = d \tan h$$

will answer for a polar dial.

NOTE FROM G. W. HILL, ESQ., TO THE EDITOR.

In Mr. Watson's article in the January Number of the Monthly, on the curve of a drawbridge, I would like to notice that the investigation could be much shortened. For, drawing vertical lines from the roller E and centre of gravity of the platform, along which lines the weights W_1 and W tend to move, and applying the principle of virtual velocities we have

$$W_1 \delta (a - r \cos \varphi) - W \delta \left(\frac{l \cos \theta}{2}\right) = 0;$$

or

$$W_1 \delta (r \cos \varphi) + W \delta \left(\frac{l ((c - r)^2 - 2 a^2)}{4 a^2}\right) = 0.$$

By integrating

$$W_1 \, r \cos \varphi + W \frac{l((c-r)^2 - 2a^2)}{4a^2} = \text{constant};$$

or, since $$\frac{2 \, W_1 a^2}{W l} = B,$$

$$2 \, B \, r \cos \varphi - 2 \, c \, r + r^2 = \text{constant},$$

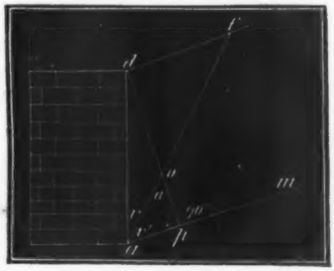
which is the equation to the curve.

Rutger's College, Jan. 22, 1859.

ON THE HORIZONTAL THRUST OF EMBANKMENTS.

By Capt. D. P. Woodbury, U. S. Corps of Engineers.

What is the horizontal thrust against a vertical wall of an embankment of homogeneous cohesive earth rising to an indefinite plane parallel to the natural slope?



Let $F =$ that thrust.

Let df, parallel to the natural slope $a\,m$, be the indefinite surface of the embankment.

Let $F' =$ the variable horizontal force which, acting at right angles to $a\,d$, shall be just sufficient to prevent the descent of any prism $a\,d\,f$ along its base $f\,a$. $Q =$ the surface $a\,d\,f$.

Let angle $d\,a\,f = v$; angle $d\,a\,m = a = 90° -$ the angle of friction.

From d, the top of the wall and foot of the slope, let dp be drawn perpendicular to the natural slope, $a\,m$. We shall find

$$F = \tfrac{1}{2} \, (dp)^2 = \tfrac{1}{2} \, h^2 \sin^2 a; \qquad (h = a\,d)$$

and the prism of greatest thrust will be the trapezoid resting on $a\,d$, and bounded by the indefinite lines $a\,m$ and df. This anomalous result may be thus proved.

The variable forces F' and Q may be resolved into the components $F' \cos v$, $Q \sin v$, perpendicular to af, and $F' \sin v$, $Q \cos v$, parallel to af. The force $F' \sin v$, acting along af upwards, must, assisted by the friction due to normal pressure on af, just equal the parallel and opposite force $Q \cos v$; that is,

(1) $$F' \times \sin v + (F' \cos v + Q \sin v) f = Q \cos v.$$

For f substitute $\cot a = \frac{\cos a}{\sin a}$, and reduce. There results

(2) $$F' = Q \times \operatorname{tang}(a - v).$$

(3) But $$Q = \tfrac{1}{2}\, ad \times df \times \sin a = \tfrac{1}{2}\, h^2 \times \frac{\sin a \sin v}{\sin (a - v)},$$

which gives $$F' = \tfrac{1}{2}\, h^2 \frac{\sin a \sin v}{\cos (a - v)}.$$

Let $$v' = a - v;\ \text{then } a - v' = v,$$

$$F' = \tfrac{1}{2}\, h^2 \sin a\, (\sin a - \cos a \operatorname{tang} v')$$

(4) $$= \tfrac{1}{2}\, h \sin a\, (h \sin a - h \cos a \operatorname{tang} v')$$

$$= \tfrac{1}{2}\, dp\, (dp - op) = \tfrac{1}{2}\, dp \times do.$$

The maximum value of $F' = F$ evidently corresponds to that direction of af in which op is zero, or $do = dp$, hence

(5) $$F = \tfrac{1}{2}\, h^2 \sin^2 a = \tfrac{1}{2}\, (dp)^2.$$

The thickness of a rectangular wall able to withstand this thrust, the curve of pressure crossing the base at one third its length from the exterior edge, is given by the equation (e = thickness of wall),

(6) $$F \times \tfrac{1}{3} h = eh \times \tfrac{1}{6} e;\ \text{giving } e^2 = 2F,\ e = h \sin a = dp.$$

We have thus far supposed the density of the earth and of the wall to be the same, that is, unity. Let ω = the weight of a cubic unit of the earth; ω' = the weight of a cubic unit of the wall. We have

(7) $$F = \tfrac{1}{2}\, \omega \times (dp)^2;\ \text{and } e = dp \sqrt{\frac{\omega}{\omega'}}.$$

The rule of the French engineers, which consists in doubling the horizontal thrust and determining the thickness of pile on the con-

dition that the resultant of the thrust thus increased, and the weight of the wall, shall pass through the exterior edge of the base, gives

$$2\,F \times \tfrac{1}{3}\,h = \tfrac{1}{2}\,h\,e^2;\ \text{or}\ e = dp\,\sqrt{\tfrac{2}{3}};$$

or, introducing ω and ω', as above,

$$(8) \qquad e = dp\,\sqrt{\frac{2\,\omega}{3\,\omega'}}.$$

The ratio of e in (8) and (7) is $\sqrt{\tfrac{2}{3}} = .8164$. That is, the "rule" does not give a sufficient thickness. Formula (4) includes the case of a perfect fluid; in which a being 90°, $\cos a$ is zero, and $\cos a \times \tan v' = o\,p = 0$, whatever be the angle v' or v.

The pressure of a perfect fluid therefore upon a vertical surface, which, the density being unity, is known to be $\tfrac{1}{2}\,h^2$, is only a particular case of a general law.

[From Walton's Collection of Mechanical Problems.]

PROBLEM IN PROJECTILES.

By Prof. Longfellow and William Walton.

Swift of foot was Hiawatha;
He could shoot an arrow from him,
And run forward with such fleetness,
That the arrow fell behind him!
Strong of arm was Hiawatha;
He could shoot ten arrows upward,
Shoot them with such strength and swiftness,
That the tenth had left the bow-string
Ere the first to earth had fallen.

Supposing Hiawatha to have been able to shoot an arrow every second, and, when not shooting vertically, to have aimed so that the flight of the arrow might have the longest range, prove that it would have been safe to bet long odds on him if entered for the Derby.

EDITORIAL REMARKS ON PROF. PARKER'S "NOTE ON THE INTERPRETATION OF ALGEBRAIC RESULTS."

Although we cannot entirely agree with Prof. Parker in his conclusions, still the question admits of discussion, as his interesting article clearly shows. Nor is Prof. Parker the only one who has ever doubted the correctness of the usual interpretation of the case in question, given by Bourdon and the many others who have used the "Problem of the Lights," or the entirely similar problem of "Points of equal Attraction," in the discussion of equations of the second degree. We know that many teachers, as well as students, have questioned it; and, what is more, those authors who have omitted the case altogether in their discussion of the problem, pretty evidently did not feel entirely satisfied with it. We are therefore much obliged to Prof. Parker for his clear and pointed statement of the issue, and beg to append the following remarks.

We remark, first, that if the origin be taken at some point O instead of A, and we put $OA = \alpha$,

O A B

$OB = \beta$, and let x denote the distance from O to the points of equal illumination; then if a and b denote the respective intensities of the lights A and B, we have

$$(1) \qquad \frac{a}{(x - \alpha)^2} = \frac{b}{(\beta - x)^2}; \text{ or}$$

$$(2) \qquad (\beta - x)^2 a = (x - \alpha)^2 b; \text{ or}$$

$$(3) \qquad x = \frac{\beta\sqrt{a} \pm \alpha\sqrt{b}}{\sqrt{a} \pm \sqrt{b}}.$$

If now we suppose that $\beta = \alpha$, that is, that the lights are together at A, then

$$x = \frac{\alpha(\sqrt{a} \pm \sqrt{b})}{\sqrt{a} \pm \sqrt{b}} = \alpha;$$

and both roots $= \alpha$ instead of *zero*, to which they would be equal

if the origin were taken at the point occupied by the lights. If, therefore, the particular hypothesis introduces impossible conditions into the problem, the fact, if indicated at all by any peculiarity of the roots, must be indicated by the presence of *equal* roots, and not because they both happen to be zero for a special origin. But the equal roots do not of themselves necessarily indicate any impossibility, and it is only by observing that the equation itself becomes impossible that we detect the fact that the hypothesis has introduced impossible conditions. When $\beta = \alpha = x$, we obtain

$$0^2 \times a = 0^2 \times b,$$

and the roots α satisfy form (2), whatever be the intensities; but the members of the equation in form (1) become unequal infinities for unequal intensities, and this is where the symbol of impossibility appears.

The fallacy consists in multiplying both sides of equation (1) by factors which become zero for the particular hypothesis; and failing to attend to this consideration led BOURDON into the oversight.

Again, if we put $y = \frac{a}{(x-\alpha)^2}$ and $y' = \frac{b}{(\beta - x)^2}$, and construct the curves corresponding to these equations, it is plain that their intersections will correspond to all possible solutions of (1). The intensity curve of the light A consists of two branches, to both of which the ordinate through the light is an asymptote; the axis of x is also an asymptote to both branches. The same remark applies to the intensity curve of the light B. The right hand branch of A's curve will cut both branches of B's, and these two are the only points of intersection and the only solutions.

When $\beta = \alpha$, the ratio of the ordinates or intensities is

$$\frac{y}{y'} = \frac{a\,(\alpha - x)^2}{b\,(x-\alpha)^2} = \frac{a}{b} = \text{constant}$$

for all values of x, including $x = \alpha$; and therefore, when both

lights occupy the same point, and B's intensity at a unit's distance is less than A's at the same distance, B's intensity curve will fall entirely within A's, and there is no intersection and no solution. When, therefore, two lights of unequal intensities occupy the same place, there is no point in space which they equally illuminate; not even the one in which they are both situated.

When the intensities are equal, that is, when $a = b$, as well as $\beta = \alpha$, the curves coincide throughout their whole extent, and this indeterminateness indicates that all points on the line are equally illuminated by the lights. But so long as there is any distance at all, however small, between the lights, the curves will cut in two points, and the problem will have two possible solutions.

Finally, we remark that the error in question does not seem to us to be one of interpretation, as Prof. PARKER supposes. It consists not in an impossible form of the roots, but in an *implicit fallacy*, to which algebraic transformations are often liable.

We must conclude, that when $x = 0$ is the true root of an equation, it does not, like infinity, indicate impossible conditions in the problem, but must be interpreted in the usual manner.

THEOREM ON RECTANGULAR COÖRDINATES.

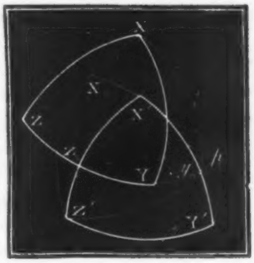
BY W. P. G. BARTLETT,
Nautical Almanac Office, Cambridge, Mass.

THE well-known equation,

$$\cos {}^{x'}_{x} = \cos {}^{y'}_{y} \cos {}^{z'}_{z} - \cos {}^{y'}_{z} \cos {}^{z'}_{y},$$

may be simply deduced as follows. Let the systems of axes intersect a sphere at $x\,y\,z$ and $x'\,y'\,z'$, the origin of each system being placed at the centre of the sphere. Let the quadrantal spherical

triangle $x\,y\,z$ be moved on the surface of the sphere into the position $x\,y_0\,z_0$, then in the same way into the position $x_0\,y_0\,z'$, and finally into the position $x'\,y'\,z'$. Let $y'\,p$ be a great circle arc perpendicular to the arc $z\,y$. Then from the various right angled triangles thus formed we get, by considering the arcs, the following relations between the angles which they represent.



$$\begin{aligned}\cos\genfrac{}{}{0pt}{}{x'}{x} &= \cos\genfrac{}{}{0pt}{}{z}{z_0}\cos\genfrac{}{}{0pt}{}{x'}{x_0} = \cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{y'}{y_0} = \cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{p}{y_0}\cos\genfrac{}{}{0pt}{}{y'}{p}\\ &= \cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{y'}{p}\cos\left(\genfrac{}{}{0pt}{}{y}{p} \backsim \genfrac{}{}{0pt}{}{y}{y_0}\right) = \cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{y'}{p}\left(\cos\genfrac{}{}{0pt}{}{y}{p}\cos\genfrac{}{}{0pt}{}{y}{y_0} + \sin\genfrac{}{}{0pt}{}{y}{p}\sin\genfrac{}{}{0pt}{}{y}{y_0}\right)\\ &= \cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{y'}{p}\left(\cos\genfrac{}{}{0pt}{}{y}{p}\cos\genfrac{}{}{0pt}{}{z}{z_0} - \cos\genfrac{}{}{0pt}{}{z}{p}\cos\genfrac{}{}{0pt}{}{y}{z_0}\right)\\ &= \cos\genfrac{}{}{0pt}{}{y'}{p}\cos\genfrac{}{}{0pt}{}{y}{p}\cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{z}{z_0} - \cos\genfrac{}{}{0pt}{}{y'}{p}\cos\genfrac{}{}{0pt}{}{z}{p}\cos\genfrac{}{}{0pt}{}{z'}{z_0}\cos\genfrac{}{}{0pt}{}{y}{z_0}\\ &= \cos\genfrac{}{}{0pt}{}{y'}{y}\cos\genfrac{}{}{0pt}{}{z'}{z} - \cos\genfrac{}{}{0pt}{}{y'}{z}\cos\genfrac{}{}{0pt}{}{z'}{y}.\end{aligned}$$

I have used a German notation, $\backsim$, to denote the difference between $\genfrac{}{}{0pt}{}{y}{p}$ and $\genfrac{}{}{0pt}{}{y}{y_0}$, because p may fall on the other side of y_0. Although I have not been able to find this demonstration, I should be surprised if it had not been given before. Can any one furnish information on the point?

Mathematical Monthly Notices.

Asteroids for the Year 1859. *A Supplement to the American Ephemeris for* 1861.

Besides the opposition Ephemerides which have been published from time to time, by the Superintendent of the American Ephemeris and Nautical Almanac for comparison with observations, this is the first regular issue of the Ephemerides of any considerable number of the Asteriod group as part of the American Ephemeris. The large number of the Asteroids already known, with their probable increase, will make the preparation of this department no insignificant part of the annual labor; and we propose therefore briefly to indicate the plan which has been adopted by Prof. Winlock for carrying on this part of the work. 1st. The same epoch and intervals of time are adopted in all the computations of Special Perturbations. 2d. The coördinates of the disturbing planets, and all that part of the labor independent of the particular Asteroid, are once for all carefully computed and checked for the adopted epoch and

intervals; thus saving the labor which must otherwise be performed for each separate Asteroid. 3d. Instead of the usual frequent correction of the elements, they will remain unchanged until the perturbations have accumulated to such a degree that it will be a saving of labor to incorporate them into a new set of corrected elements; and in the mean time any small corrections of the elements which may seem desirable will be combined with the perturbations. In this way a tolerably good set of elements will not probably need correction and change of epoch short of five thousand days. 4th. The computations of the Special Perturbations of all the Asteroids to which special methods are applied will be carried on simultaneously for the same dates, a labor-saving arrangement which no one but an experienced practical astronomer can fully appreciate. It gives us great pleasure to record this evidence of Prof. WINLOCK's able and judicious superintendence of the American Ephemeris.

This Supplement contains the Ephemerides of thirty-three of the fifty-six Asteroids, with Tables of elements and authorities. Almost the entire history of the Asteroid group, except a column of relative brightness to be added hereafter, is contained in the following pages, which we have thought it desirable to extract to save the readers of the Monthly the trouble of other reference; and for the same reason we add the following definitions of symbols. The Ecliptic is the plane of reference, which the plane of the orbit cuts in a straight line, called "The line of the Nodes."

The longitude of the Ascending node, or point through which the asteroid passes from south to north of the ecliptic, denoted by ☊, is the angular distance of this point from the vernal equinox, or first point of Aries.

The inclination of the plane of the orbit to that of the ecliptic is denoted by i, and the two elements ☊ and i fix the position of the plane of the orbit

The longitude of the perihelion, denoted by π, fixes the position of the orbit in its own plane, and is counted on the Ecliptic from the first point of Aries to the ascending node, then on the plane of the orbit in the direction of the Asteroid's motion until we reach its perihelion, or point of least distance from the sun.

The mean distance from the sun is denoted by a, the eccentricity by e, and these two elements determine the size and shape of the orbit. The mean orbit longitude of the Asteroid for the epoch, denoted by L, is counted on the Ecliptic from the first point of Aries to the Ascending node, then on the plane of the orbit in the direction of the Asteroid's motion until we arrive at its place. The mean daily motion is denoted by μ, which enables us to find the mean orbit longitude for any date before or after the epoch. If one imagines himself standing at the Sun on the north side of the Ecliptic, the angles are counted from right to left, that is, towards the east, and the motions of the Asteroids are in the same direction.

The Asteroids numbered (41) and (41)* have this interesting history: In 1856, May 23, Dr. Goldschmidt discovered Daphne. It was at this time near its conjunction, and was therefore lost in the sun's rays before any thing more than the first rough approximation to its orbit could be obtained. In 1857, September 9, he again observed what he supposed to be the lost Daphne; but Mr. Schubert has shown that the two sets of observations do not correspond to the same orbit, and therefore that the Daphne of 1856 is still a "missing star."

The following pages, on which are found the elements with authorities, are printed on duplicates of the electrotype plates prepared for this Supplement, for which we are indebted to the courtesy of Prof. WINLOCK.

① *Ceres.* — *Astronomical Journal*, Vol. III. p. 165, by Mr. ERNEST SCHUBERT, from a thorough discussion of observations from 1832 to 1853, taking account of perturbations by Jupiter only. They have been reduced by him from 1854, January 0, to 1859, September 7, by applying the perturbations depending on Jupiter and Saturn. Comparison with observations at opposition in 1858 gave $\Delta\alpha \cos\delta = -5''.2$, $\Delta\delta = +6''.2$.

② *Pallas.* — *English Nautical Almanac* for 1860, p. 572, by Mr. FARLEY, from eight oppositions, 1845 to 1853, inclusive, reduced, by addition of perturbations, depending on Venus, the Earth, Mars, Jupiter, and Saturn, to 1858, May 29, Greenwich. They nearly satisfy all the observations made at Greenwich near the times of oppositions as far as 1855 inclusive.

③ *Juno.* — *English Nautical Almanac* for 1859, p. 564, from twelve oppositions, 1841 to 1855 inclusive, reduced by addition of perturbations depending on Venus, the Earth, Mars, Jupiter, and Saturn. Comparison with Greenwich observations at opposition in 1856 gave $\Delta\alpha \cos\delta = -10''.7$, $\Delta\delta = +0''.7$, and at Königsberg in 1858,

$$\Delta\alpha \cos\delta = -21''.0, \quad \Delta\delta = +3''.0.$$

④ *Vesta.* — *English Nautical Almanac* for the year 1860, p. 575, by Mr. FARLEY, from twelve oppositions, 1840 to 1855 inclusive, reduced by addition of perturbations depending on Venus, the Earth, Mars, Jupiter, and Saturn. They very nearly satisfy all the observations made at Greenwich near the times of oppositions as far as 1855 inclusive, and observations at Königsberg in 1858, within about 5″.

⑤ *Astræa.* — *Berliner Astron. Jahrbuch* for the year 1858, by Professor ZECH. They have satisfied observations at seven oppositions, from 1845 to 1853 inclusive, and at the opposition in 1856 gave, about, $\Delta\alpha \cos\delta = +13''$, $\Delta\delta = +4''$.

⑥ *Hebe.* — *Astronomische Nachrichten*, Vol. XXXI. p. 13, by R. LUTHER, from four oppositions, 1847 - 1850; in 1857 the errors at opposition were $\Delta\alpha \cos\delta = +21''$, $\Delta\delta = -7''$.

⑦ *Iris.* — *Astronomische Nachrichten*, Vol. XXVIII. p. 277, by Mr. ERNEST SCHUBERT, from two oppositions, 1847 - 1848, reduced by addition of perturbations. They have agreed with observations since, until 1858, when the errors were

$$\Delta\alpha \cos\delta = 46'', \quad \Delta\delta = 15''.$$

⑧ *Flora.* — *Tables of Flora*, by Professor F. BRÜNNOW, Berlin, 1855. They were computed from four oppositions, 1848 - 1852.

⑨ *Metis.* — *Astronomische Nachrichten*, Vol. XXXVI. p. 71, by J. PH. WOLFERS, from six oppositions, 1848 - 1852. Have agreed with observations since; at opposition in 1857 the errors were $\Delta\alpha \cos\delta = -11''$, $\Delta\delta = -1''$.

⑩ *Hygea.* — *Astronomische Nachrichten*, Vol. XXXIX. p. 347, by Professor J. ZECH, from five oppositions, 1849 - 1854, reduced by addition of perturbations. At opposition in 1856 the errors were $\Delta\alpha \cos\delta = -8''$, $\Delta\delta = +1''$.

⑪ *Parthenope.* — *Astronomische Nachrichten*, Vol. XLI. p. 283, from four oppositions, 1850 - 1854. Errors in 1857, $\Delta\alpha \cos\delta = -3''$, $\Delta\delta = -6''$.

⑫ *Clio.* — *Astronomische Nachrichten*, Vol. XLV. p. 321, by Professor F. BRÜNNOW, from six oppositions, 1850 - 1856. Tables have been constructed by him.

⑬ *Egeria.* — *Astronomical Journal*, Vol. II. p. 282, by Professor J. S. HUBBARD, 1850 - 1851. Tables have been constructed by Professor PEIRCE.

⑭ *Irene.* — *Astronomische Nachrichten*, Vol. XLII. p. 141, from four oppositions, 1851–1855, by C. Bruhns. At opposition in November, 1857, the errors were

$$\Delta\alpha \cos\delta = -4'', \quad \Delta\delta = -1''.$$

⑮ *Eunomia.* — *Astronomical Journal*, Vol. IV. p. 170, by Mr. Ernest Schubert, from four oppositions, 1851–1854. Have agreed well with observations since. At opposition in 1858 the errors were $\Delta\alpha \cos\delta = +3''$, $\Delta\delta = -3''$.

⑯ *Psyche.* — Provisional elements selected, and reduced by Mr. Schubert by addition of perturbations preparatory to a new determination of the orbit.

⑰ *Thetis.* — *Berliner Astron. Jahrbuch*, 1859, p. 419, by E. Schönfeld, from four oppositions, 1852–1856. The errors at opposition in 1857 were $\Delta\alpha \cos\delta = -38''$, $\Delta\delta = -13''$.

⑱ *Melpomene.* — *Astronomical Journal*, Vol. V. p. 41, from four oppositions, 1852–1856. At opposition in 1858, $\Delta\alpha \cos\delta = +6''$, $\Delta\delta = -3''$.

⑲ *Fortuna.* — *Astronomische Nachrichten*, Vol. XLVI. p. 247, by C. Powalky, from four oppositions, 1852–1856. Errors at opposition in 1858, $\Delta\alpha \cos\delta = -10''$, $\Delta\delta = +5''$.

⑳ *Massilia.* — *Astronomische Nachrichten*, Vol. XLV. p. 287, by W. Günther, from four oppositions, 1852–1856, perturbations by Jupiter alone being applied. In 1858,

$$\Delta\alpha \cos\delta = -11'', \quad \Delta\delta = +1''.$$

㉑ *Lutetia.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 17, from four oppositions, perturbations by Jupiter alone being taken account of. Errors at opposition in 1858,

$$\Delta\alpha \cos\delta = +7'', \quad \Delta\delta = +1''.$$

㉒ *Calliope.* — Vienna *Sitzungsberichte*, 1855, by Dr. C. Hornstein, corrected by T. H. Safford, Jr., so as to satisfy four oppositions, 1852–1856.

㉓ *Thalia.* — *Astronomical Journal*, Vol. V. p. 107, by Ernest Schubert, from four oppositions, 1853–1856. Errors at opposition in 1858, $\Delta\alpha \cos\delta = +4''$, $\Delta\delta = +1''$.

㉔ *Themis.* — *Astronomische Nachrichten*, Vol. XLVII. p. 161, by Dr. A. Krüger, from four oppositions, 1853–1856. At opposition in 1858, $\Delta\alpha \cos\delta = +2''$, $\Delta\delta = -1''$.

㉕ *Phocæa.* — *Astronomische Nachrichten*, Vol. XLVI. p. 129, by W. Günther, from three oppositions, 1853–1856. Errors in 1857, $\Delta\alpha \cos\delta = +19''$, $\Delta\delta = -7''$.

㉖ *Proserpina.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 171, by J. A. C. Oudemanns, corrected by M. Hoek to satisfy four oppositions, 1853–1857. Errors at the opposition in 1858, $\Delta\alpha \cos\delta = +14''$, $\Delta\delta = -2''$.

㉗ *Euterpe* — *Astronomische Nachrichten*, Vol. XLVIII. p. 229, by W. Günther, from four oppositions, 1853–1858.

㉘ *Bellona.* — *Berliner Astron. Jahrbuch*, 1859, from two oppositions, 1854–1855. They have not been compared with observations since.

㉙ *Amphitrite.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 363, by W. Günther, from four oppositions, 1854–1858.

㉚ *Urania.* — *Astronomische Nachrichten*, Vol. XLVII. p. 21, by W. Günther, from three oppositions, 1854–1857.

㉛ *Euphrosyne.* — *Astronomische Nachrichten*, Vol. XLI. p. 289, by A. Winnecke, from one opposition, 1854–1855. Errors at opposition in 1857 were $\Delta\alpha \cos\delta = +1''$, $\Delta\delta = +10''$.

(32) *Pomona.* — Elements selected and reduced by Mr. ERNEST SCHUBERT, preparatory to a new determination of the orbit.

(33) *Polyhymnia.* — Selected for corrrection by Mr. SCHUBERT.

(34) *Circe.* — *Berliner Astron. Jahrbuch*, 1859, p. 420, from two oppositions, 1855 – 1856, by Dr. W. KLINKERFUES. At opposition in 1857, the errors were,

$$\Delta\,\alpha \cos\delta = -14'\,3'', \quad \Delta\,\delta = -3'\,45''.$$

(35) *Leucothea.* — Selected for correction by Mr. ERNEST SCHUBERT.

(36) *Atalanta.* — *Berliner Astron. Jahrbuch*, 1860, p. 404, from two oppositions, by Dr. W. FÖRSTER, 1855 – 1857; agreed well with observation in 1858.

(37) *Fides.* — *Astronomische Nachrichten*, Vol. XLV. p. 17, from one opposition, by G. RÜMKER, 1855 – 1856; in 1857 they were in error about 20″ in R. A. and 14″ in Dec.

(38) *Leda.* — *Berliner Astron. Jahrbuch*, 1860, from one opposition, 1856, by M. LÖWY; agreed with observation in 1858 within about 2′ in R. A. and 1′ in Dec.

(39) *Lætitia.* — *Astronomische Nachrichten*, Vol. XLV. p. 379, from one opposition, 1856, by M. ALLÉ.

(40) *Harmonia.* — *Astronomische Nachrichten*, Vol. XLIV. p. 281, from one opposition, 1856, by C. POWALKY. Did not agree well with observation in 1857.

(41) *Daphne.* — *Astronomische Nachrichten*, Vol. XLVII. p. 26, from five days' observations by C. F. PAPE, very uncertain.

(41)* *Astronomical Journal*, Vol. V. p. 174, by Mr. ERNEST SCHUBERT, from observations in 1857.

(42) *Isis.* — *Astronomische Nachrichten*, Vol. XLVI. p. 91, from observations in 1856. In December, 1857, the errors were $\Delta\,\alpha = -1'.7$, $\Delta\,\delta = -0'.6$.

(43) *Ariadne.* — *Astronomische Nachrichten*, Vol. XLIX. p. 39, by E. WEISS, from observations in 1857.

(44) *Nysa.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 233, by M. GUSSEW, from observations in 1857.

(45) . . . . *Astronomische Nachrichten*, Vol. XLVIII. p. 359, by M. LÖWY, from observations in 1857.

(46) *Hestia.* — *Astronomical Journal*, Vol. V. p. 153, by J. C. WATSON.

(47) *Aglaia.* — From observations in 1857, by T. H. SAFFORD, Jr. In February, 1858, the errors were $\Delta\,\alpha = +50''$, $\Delta\,\delta = +20''$.

(48) *Doris.* — *Astronomische Nachrichten*, Vol. XLVII. p. 319, by C. POWALKY.

(49) *Pales.* — *Astronomische Nachrichten*, Vol. XLVII. p. 315, by C. POWALKY.

(50) *Verginia.* — *Astronomical Journal*, Vol. V. p. 118, by Mr. JAMES FERGUSON. They will probably give the place of the planet within 5′.

(51) *Nemausa.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 124, from a few observations, by Dr. W. FÖRSTER.

(52) *Europa.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 221, by Dr. H. S. SCHULTZ. Approximate.

(53) *Calypso.* — *Astronomische Nachrichten*, Vol. XLVIII. p. 335, by W. OELTZEN.

(54) . . . . *Astronomische Nachrichten*, Vol. XLIX. p. 185, by SCHJELLERUP.

(55) . . . . *Astronomical Journal*, Vol. V. p. 162, by T. H. SAFFORD, Jr. and S. NEWCOMB.

Symbol.	Name.	π.	☊.	ϕ.	i.	μ.	L.
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
①	Ceres.	149 26 13.1	80 49 54.7	4 36 12.1	10 36 27.8	12 51.3333	346 48 15.4
②	Pallas.	122 7 38.4	172 38 32.7	13 50 57.1	34 42 29.8	12 49.4780	224 28 25.5
③	Juno.	54 0 55.8	170 58 22.0	14 50 35.7	13 3 9.8	13 33.8848	104 2 31.1
④	Vesta.	250 35 29.4	103 21 10.3	5 10 31.2	7 8 9.1	16 17.8432	218 26 1.1
⑤	Astræa.	134 35 35.7	141 24 48.5	10 57 8.3	5 19 35.2	14 17.9486	80 56 2.7
⑥	Hebe.	15 2 23.4	138 35 19.5	11 38 1.9	14 46 35.4	15 39.3481	124 54 18.6
⑦	Iris.	41 29 15.3	259 46 16.1	13 20 45.9	5 28 1.4	16 2.6335	322 34 38.8
⑧	Flora.	32 54 28.3	110 17 48.6	9 0 56.3	5 53 8.0	18 6.3310	68 48 32.0
⑨	Metis.	71 3 55.6	68 31 31.6	7 5 1.6	5 36 0.6	16 2.8856	128 8 12.7
⑩	Hygea.	227 47 58.8	287 38 34.2	5 46 16.6	3 47 9.3	10 34.8491	354 47 47.6
⑪	Parthenope.	316 10 7.1	125 3 41.1	5 40 30.3	4 36 57.9	15 23.7824	283 56 41.9
⑫	Clio.	301 39 24.7	235 34 41.7	12 38 44.1	8 23 19.4	16 35.8341	7 42 5.0
⑬	Egeria.	119 12 59.0	43 17 55.7	4 52 7.4	16 33 6.7	14 18.3861	138 44 42.6
⑭	Irene.	179 28 21.9	86 40 4.5	9 30 38.1	9 7 7.4	14 11.5608	67 12 20.6
⑮	Eunomia.	27 31 8.1	293 56 15.8	10 47 54.8	11 43 39.0	13 45.2220	238 54 5.1
⑯	Psyche.	13 16 14.8	150 35 34.0	7 42 49.7	3 4 6.5	11 50.0987	50 51 42.0
⑰	Thetis.	259 22 51.2	125 27 13.3	7 17 18.4	5 35 40.7	15 11.9760	210 1 24.3
⑱	Melpomene.	15 11 48.0	150 4 33.3	12 32 14.8	10 8 58.3	16 59.8395	304 33 25.3
⑲	Fortuna.	30 22 50.2	211 29 28.7	9 5 10.8	1 32 28.8	15 30.1578	148 28 55.8
⑳	Massilia.	98 28 37.6	206 41 27.6	8 15 42.3	0 41 7.3	15 48.7396	195 16 53.9
㉑	Lutetia.	327 2 45.2	80 27 23.3	9 19 32.1	3 5 11.1	15 33.5610	41 24 9.0
㉒	Calliope.	58 16 41.1	66 36 54.7	5 56 53.6	13 44 51.9	11 54.9070	76 59 2.0
㉓	Thalia.	123 58 40.6	67 38 34.4	13 23 56.7	10 13 13.6	13 52.4617	280 7 33.7
㉔	Themis.	137 54 9.7	36 10 30.3	6 44 53.0	0 49 1.8	10 34.6753	30 2 41.5
㉕	Phocæa.	302 46 9.0	214 4 54.6	14 37 38.8	21 35 53.6	15 53.6780	294 46 13.5
㉖	Proserpina.	235 17 26.8	45 53 14.6	5 1 15.7	3 35 40.3	13 39.6815	181 21 20.9
㉗	Euterpe.	87 39 0.0	93 44 45.0	9 57 22.5	1 35 31.1	16 26.6260	260 43 32.7
㉘	Bellona.	122 22 48.3	144 43 5.4	8 53 17.5	9 22 30.8	12 47.4862	159 3 36.8
㉙	Amphitrite.	56 39 6.6	356 26 51.8	4 9 3.1	6 7 49.6	14 28.8694	293 11 23.8
㉚	Urania.	31 23 24.7	308 13 46.3	7 18 22.7	2 5 56.9	16 16.0689	19 30 24.4
㉛	Euphrosyne.	93 51 6.6	31 25 23.0	12 28 29.8	26 25 12.4	10 32.8031	53 49 50.3
㉜	Pomona.	193 33 42.5	220 48 1.4	4 37 26.6	5 28 49.1	14 11.7238	134 30 20.0
㉝	Polyhymnia.	340 51 46.1	9 16 9.2	19 41 36.4	1 56 41.5	12 10.8833	266 47 55.8
㉞	Circe.	149 58 35.1	184 47 10.8	6 12 52.4	5 26 33.2	13 24.9883	193 36 37.2
㉟	Leucothea.	198 51 53.9	355 57 26.3	12 46 9.3	8 12 10.7	11 29.3084	173 36 11.3
㊱	Atalanta.	42 22 25.0	359 8 48.4	17 19 53.4	18 42 9.5	12 58.6000	36 19 53.2
㊲	Fides.	66 5 35.8	8 10 23.4	10 4 0.8	3 7 19.3	13 46.2860	42 34 30.3
㊳	Leda.	100 40 28.4	296 27 47.3	8 57 0.8	6 58 31.9	13 2.4484	112 55 7.2
㊴	Lætitia.	1 58 57.7	157 19 31.0	6 22 38.2	10 20 50.7	12 49.8940	146 44 19.7
㊵	Harmonia.	2 1 50.9	93 32 2.9	2 38 29.0	4 15 48.4	17 19.4100	222 12 9.1

Symbol.	Period.	a.	e.	Epoch.	Date of Discovery.	By whom Discovered.
	d					
1	1680.207	2.765938	0.080257	1859, Sept. 7.0000	1801, Jan. 1	Piazzi, at Palermo.
2	1684.258	2.770386	0.239367	1858, May 28.7860	1802, Mar. 28	Harding, at Göttingen.
3	1592.365	2.668678	0.256176	1858, Jan. 28.7860	1804, Sept. 1	Olbers, at Bremen.
4	1325.366	2.361339	0.090204	1858, April 22.7860	1807, Mar. 29	Olbers, at Bremen.
5	1510.580	2.576500	0.189992	1849, Dec. 30.7488	1845, Dec. 8	Hencke, at Driessen.
6	1379.680	2.425418	0.201657	1857, Feb. 12.7488	1847, July 1	Hencke, at Driessen.
7	1346.307	2.386147	0.230832	1858, July 18.7488	1847, Aug. 13	Hind, at London.
8	1193.007	2.201386	0.156704	1848, Jan. 0.7488	1847, Oct. 18	Hind, at London.
9	1345.954	2.385730	0.123321	1858, June 29.7488	1848, April 25	Graham, at Markree.
10	2041.430	3.149373	0.100557	1851, Sept. 16.7488	1849, April 12	De Gasparis, at Naples.
11	1402.928	2.452588	0.098887	1858, June 26.7488	1850, May 13	De Gasparis, at Naples.
12	1301.423	2.332811	0.218920	1850, Dec. 30.7488	1850, Sept. 13	Hind, at London.
13	1509.810	2.575625	0.084873	1851, Dec. 5.0000	1850, Nov. 2	De Gasparis, at Naples.
14	1521.912	2.589368	0.165230	1857, Nov. 19.7488	1851, May 20	Hind, at London.
15	1570.486	2.644180	0.187357	1859, May 11.0000	1851, July 29	De Gasparis, at Naples.
16	1825.098	2.922752	0.134225	1860, Nov. 20.0000	1852, Mar. 17	De Gasparis, at Naples.
17	1421.090	2.473710	0.126865	1856, April 3.7488	1852, April 17	Luther, at Bilk.
18	1270.788	2.296060	0.217078	1859, July 2.0000	1852, June 24	Hind, at London.
19	1393.312	2.441368	0.157922	1858, Mar. 2.7488	1852, Aug. 22	Hind, at London.
20	1366.023	2.409386	0.143696	1858, April 20.7488	1852, Sept. 19	Chacornac, at Marseilles.
21	1388.232	2.435431	0.162045	1853, Jan. 1.7488	1852, Nov. 15	Goldschmidt, at Paris.
22	1439.977	2.495579	0.103630	1852, Dec. 30.7488	1852, Nov. 16	Hind, at London.
23	1556.829	2.628824	0.231732	1859, July 10.0000	1852, Dec. 15	Hind, at London.
24	2041.989	3.149947	0.117504	1856, Sept. 24.7488	1853, April 5	De Gasparis, at Naples.
25	1358.949	2.401060	0.252533	1857, July 9.7488	1853, April 6	Chacornac, at Marseilles.
26	1581.102	2.656079	0.087521	1857, Mar. 19.7488	1853, May 5	Luther, at Bilk.
27	1313.568	2.347305	0.172896	1859, June 13.7488	1853, Nov. 8	Hind, at London.
28	1688.630	2.775177	0.154507	1854, Feb. 27.7488	1854, May 1	Luther, at Bilk.
29	1491.594	2.554866	0.072383	1859, July 8.7488	1854, Mar. 1	Luther, at Bilk.
30	1327.805	2.364199	0.127174	1858, Oct. 8.7488	1854, July 22	Hind, at London.
31	2048.030	3.156158	0.216013	1854, Dec. 30.7488	1854, Sept. 1	Ferguson, at Washington
32	1521.620	2.589039	0.080617	1860, Jan. 24.7488	1854, Oct. 26	Goldschmidt, at Paris.
33	1773.197	2.867075	0.336987	1858, April 13.7488	1854, Oct. 28	Chacornac, at Paris.
34	1609.961	2.688302	0.108253	1855, April 9.4488	1855, April 15	Chacornac, at Paris.
35	1880.145	2.981229	0.221025	1860, Feb. 14.0000	1855, April 19	Luther, at Bilk.
36	1664.526	2.748705	0.297900	1855, Dec. 30.7488	1855, Oct. 5	Goldschmidt, at Paris.
37	1568.465	2.641907	0.174798	1855, Dec. 30.7488	1855, Oct. 5	Luther, at Bilk.
38	1656.339	2.739685	0.155576	1855, Dec. 30.7488	1856, Jan. 12	Chacornac, at Paris.
39	1683.349	2.769387	0.111075	1855, Dec. 31.7488	1856, Feb. 8	Chacornac, at Paris.
40	1246.861	2.267148	0.046085	1856, June 30.7488	1856, Mar. 31	Goldschmidt, at Paris.

Symbol.	Name.	π.	☊.	φ.	i.	μ.	L.
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
(41)	Daphne.	230 21 29.8	180 5 50.8	11 40 57.0	15 48 23.0	15 54.1100	202 28 48.5
(41)*	. . . . .	303 17 28.1	195 29 38.4	11 42 3.8	7 38 19.1	14 40.0100	335 48 51.5
(42)	Isis.	317 57 48.4	84 27 49.7	12 52 50.1	8 34 39.6	15 34.4490	276 45 1.9
(43)	Ariadne.	277 14 9.5	264 29 27.4	9 38 46.6	3 27 47.6	18 4.5177	224 5 10.4
(44)	Nysa.	111 46 12.3	130 54 33.4	8 25 51.6	3 41 56.6	15 36.4700	232 55 23.7
(45)	. . . . .	235 4 34.4	147 51 37.7	4 54 10.7	6 35 59.1	13 5.1037	215 29 8.3
(46)	Hestia.	355 4 36.8	181 26 43.6	9 45 28.8	2 17 48.3	14 36.5246	333 1 31.1
(47)	Aglaia.	314 16 26.4	4 29 19.6	7 21 42.5	5 0 24.7	12 5.8040	0 37 45.4
(48)	Doris.	77 11 47.7	185 13 39.9	4 25 19.8	6 29 44.0	10 47.9290	359 3 37.3
(49)	Pales.	32 49 23.3	290 27 1.0	13 44 54.4	3 8 25.0	10 54.4680	10 29 28.9
(50)	Verginia.	10 29 59.0	173 30 22.8	16 41 14.6	2 47 45.7	13 42.0410	12 5 7.9
(51)	Nemausa.	190 12 40.0	175 37 43.9	3 36 13.0	10 14 39.4	16 7.6380	172 47 1.8
(52)	Europa.	102 10 43.7	129 55 43.8	5 52 11.5	7 23 48.7	10 50.6371	147 35 49.8
(53)	Calypso.	94 38 52.3	143 30 27.8	10 23 3.6	5 3 38.8	14 0.0860	169 59 43.1
(54)	. . . . .	306 19 28.9	313 22 43.9	10 50 23.7	11 31 21.0	13 9.0720	329 25 3.3
(55)	. . . . .	21 47 23.8	10 51 28.2	7 41 19.4	7 36 47.4	12 46.0760	10 49 0.2

PERIODIC COMETS.

Name.	π.	☊.	φ.	i.	μ.	L.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
Halley's.	304 32 16.6	55 10 43.7	75 19 40.2	162 14 54.9	0 46.5067	304 32 16.6
Encke's.	157 57 30.0	334 28 34.0	(57 57 30.3)	13 4 15.0	17 54.0500	157 59 18.0
Biela's I.	108 58 52.7	245 54 5.2	49 7 23.6	12 33 49.6	8 55.2767	108 58 52.7
Biela's II.	109 5 56.0	245 50 9.9	49 2 34.5	12 33 27.8	8 58.7065	109 5 56.0
Faye's.	49 49 4.6	209 45 23.4	33 42 43.4	11 21 36.7	7 55.1849	49 49 4.6
Brorsen's.	115 43 44.4	101 46 41.7	53 21 5.6	29 48 59.2	10 37.9355	115 43 44.4
Winnecke's.	275 59 53.0	113 0 53.1	47 35 5.2	10 42 43.4	11 48.0070	275 59 33.3
Tuttle's.	115 51 35.0	269 3 13.0	55 10 31.4	54 24 10.5	4 18.9576	116 10 44.5

Symbol.	Period.	a.	e.	Epoch.	Date of Discovery.	By whom Discovered.
	d					
41	1358.334	2.400337	0.202488	1856, May 31.2488	1856, May 23	Goldschmidt, at Paris.
41*	1472.710	2.533257	0.202805	1857, Sept. 16.2844	1857, Sept. 9	Goldschmidt, at Paris.
42	1386.913	2.433889	0.222920	1856, June 30.7488	1857, May 23	Pogson, at Oxford.
43	1195.001	2.203838	0.167565	1857, April 16.7488	1857, April 15	Pogson, at Oxford.
44	1383.921	2.430386	0.146618	1857, July 9.7488	1857, May 27	Goldschmidt, at Paris.
45	1659.191	2.742828	0.085469	1856, Dec. 30.7488	1857, June 27	Goldschmidt, at Paris.
46	1478.567	2.539968	0.169487	1857, Sept. 19.5000	1857, Aug. 16	Pogson, at Oxford.
47	1785.606	2.880435	0.128134	1857, Nov. 16.0000	1857, Sept. 15	Luther, at Bilk.
48	2000.219	3.106845	0.077105	1857, Oct. 30.7488	1857, Sept. 19	Goldschmidt, at Paris.
49	1980.234	3.086115	0.237660	1857, Oct. 30.7488	1857, Sept. 19	" "
50	1576.563	2.650994	0.287150	1857, Oct. 5.0000	1857, Oct. 4	Ferguson, at Washington.
51	1339.344	2.377912	0.062853	1858, Mar. 2.3400	1858, Jan. 22	Laurent, at Nismes.
52	1991.893	3.098218	0.102270	1858, Mar. 3.2052	1858, Feb. 4	Goldschmidt, at Paris.
53	1542.700	2.612894	0.180250	1858, April 27.2635	1858, April 4	Luther, at Bilk.
54	1642.436	2.724332	0.188066	1858, Sept. 25.1342	1858, Sept. 11	Goldschmidt, at Paris.
55	1691.750	2.778581	0.133791	1858, Sept. 27.3496	1858, Sept. 11	Searle, at Albany.

PERIODIC COMETS.

Period.	a.	e.	Epoch.	Perihelion Passage.
d				
27866.953	17.988470	0.967391	1835, Nov. 15.6941	1912.1
1206.648	2.218135	0.847663	1858, Oct. 18.2488	1862.1
2421.174	3.528733	0.756119	1852, Sept. 22.7316	1859.4
2405.760	3.513750	0.755201	1852, Sept. 23.4975	1859.4
2727.360	3.820286	0.555020	1858, Sept. 12.3908	1866.2
2031.554	3.139206	0.802313	1857, Mar. 20.0128	1862.8
1830.490	2.928505	0.738276	1858, May 2.2488	1863.3
5004.680	5.726007	0.820904	1858, Feb. 27.7488	1871.9

Editorial Items.

Asaph Hall, Esq., Assistant at Harvard College Observatory, says: "I have found the following erratum in 'Theoria Motus,' Art. 83, page 108 of Davis's translation. In the denominator of the right-hand member of the last equation in the Article, for cot $\frac{1}{2}(N''-N)$ read cot $\frac{1}{2}(N''-N')$. In an erratum previously sent by Mr. Hall, ν is misprinted for χ. ..... George W. Jones, of the Senior Class of Yale College, solved all the prize problems in the October number of the Monthly, instead of George W. Fisher, whose name was printed by mistake...... It gives us pleasure to add the following names to our list of co-operators and contributors: — W. LeRoy Brown, Principal of Bloomfield Academy, Joy Depot, Albemarle Co., Va. Gerardus B. Docharty, LL.D., Professor of Mathematics in the New York Free Academy. J. H. Good, Professor of Mathematics in Heidelberg College, Tiffin, Seneca Co., Ohio. John F. Lanneau, of Furman University, Greenville, S. C...... In our last Number we gave the Prize Problem in the "Lady's and Gentleman's Diary" for 1859, hoping that some of our readers might feel sufficient interest in it to compete for the prize. Mr. Simon Newcomb, Assistant upon the American Ephemeris, and Mr. George B. Vose, Assistant in the U. S. Coast Survey, Washington, D. C., have sent us solutions, and we should like permission from these gentlemen to communicate their solutions to Prof. W. S. B. Woolhouse, the Editor of the Diary...... Note from Rev. T. W. Higginson to the Editor: —

"Worcester, Mass., January 14, 1859.

"Dear Sir: — The enclosed note found its way into print without my intending or expecting it; but I thought you might like to see it, as evidence of the value which we, in this direction, attach to your Magazine. I wish that the same thing could be done in other places; for I am satisfied that there are in many of our High Schools (certainly in this one), young mathematicians, both male and female, who can solve the easier portion of your prize problems.

"Truly yours,

"T. W. Higginson."

"Liberal offer to the High School. — We have been permitted to publish the following letter, addressed to Mr. Calkins, Principal of the Mathematical Department of the High School. It is gratifying to record such evidence of the interest our distinguished citizens are taking in this noble science: —

'Worcester, January 9, 1859.

'Dear Sir: — You have no doubt observed the prize problems in the Mathematical Monthly, and have formed an opinion as to whether your pupils could perform many of them. If you think it worth while, I would gladly promise a bound copy of Vol. I. of the Monthly to the member of your school sending the best solutions of the largest number of problems between this time and the close of the volume. This might be an additional incentive to some who distrusted their ability to gain Mr. Runkle's prizes.

'It would be an especial gratification to me if I could thus secure some female names among the monthly lists of those solving the problems.

'Cordially yours,

'T. W. Higginson.'"

This evidence of interest in the Monthly is deeply gratifying, and we are ready to co-operate in all possible ways to carry out the suggestion of Mr. HIGGINSON. If the teachers of Mathematics in our High Schools and Academies think our Prize Problems are too difficult, and will communicate those they think of the proper degree of difficulty, we will gladly insert five of them each month, entitled *Prize Problems for Students in Academies, High and Normal Schools.* The teachers and friends of each Institution can then offer such prizes, with such conditions, as they see fit. It would, however, be exceedingly desirable for each Prize Committee to report to us at the end of the year; as we should then be able to show the relative standard of such institutions in regard to mathematical instruction, as well as announce the name of the student who had solved the largest number of the prize problems.

BOOKS RECEIVED. — Cours de Mécanique Appliquée; par M. MAHISTRE. Paris: Mallet-Bachelier, 1858. — Lecons de Mécanique Élémentaire a l'usage Des Candidats à l'École Polytechnique et à l'École Normale supérieure; par M. OSSIAN BONNET. Première Partie. Paris: Mallet-Bachelier, 1858. — Encyclopédie Mathématique ou exposition complète de toutes des Mathématiques d'après les principes de la Philosophie des Mathématiques de Hoëné Wronski; par A. S. DE MONTFERRIER. Tomes 1, 2, 3. To be continued in monthly parts. — Physique a l'usage des Gens du Monde, 308 Magnifiques Vignettes. 12 mo.; par GANOT. Cours de Physique de l'École Polytechnique. 8vo. JARMIN. — Construction of Wrought-Iron Bridges; by LATHAM. — Manual of applied Mechanics; by RANKINE. — Quarterly Journal of Pure and Applied Mathematics for Nov. 1858. — No. 1, of Vol. III. Nouvelles Annales de Mathématiques, Decembre, 1858.

MATHEMATICAL PAPERS *published by the Smithsonian Institution, in the "Smithsonian Contributions to Knowledge."*

Vol. II. Article I. Researches relative to the Planet Neptune; by SEARS C. WALKER, Esq., pp. 60.

Appendix I. Ephemeris of the Planet Neptune for the date of the Lalande observations of May 8 and 10, 1795, and for the oppositions of 1846, 1847, 1848, and 1849, pp. 32. II. Ephemeris of the Planet Neptune for the years 1850 and 1851; by SEARS C. WALKER, Esq., pp. 20. III. Occultations visible in the United States during the year 1851; computed by JOHN DOWNES, Esq., pp. 26.

Vol. III. Article II. Researches on Electrical Rheometry; by A. SECCHI, pp. 60, and three plates.

Appendix I. Ephemeris of the Planet Neptune for the year 1852; by SEARS C. WALKER, Esq., pp. 10. II. Occultations visible in the United States, and other parts of the world during the year 1852; computed by JOHN DOWNES, Esq., pp. 36.

Vol. VIII. Article IV. The Tangencies of Circles and of Spheres; by BENJAMIN ALVORD, Major U. S. Army, pp. 16, and nine plates.

Vol. IX. Article II. On the Relative Intensity of the Heat and Light of the Sun upon different latitudes of the earth; by L. W. MEECH, pp. 58, and six plates.

Appendix. New Tables for determining the values of the Coefficients in the Perturbative Function of Planetary Motion, which depend upon the ratio of the mean distances; by JOHN D. RUNKLE, Assistant in the office of the American Ephemeris and Nautical Almanac, pp. 64. Asteroid Supplement to New Tables for determining the values of $b_s^{(i)}$ and its derivatives; by JOHN D. RUNKLE, pp. 72.

Besides the above, the Appendices to Vols. VIII. and IX. contain a list of the "Publications of Learned Societies and Periodicals in the Library of the Smithsonian Institution. Part I.

pp. 40; part II. pp. 38." These Appendices show the very great value of this department of the Library, and it is gratifying to observe that almost the whole of this invaluable collection has been donated to the Smithsonian Institution by the various Societies.

MATHEMATICAL PAPERS *published by the American Academy of Arts and Sciences, in the New Series, commenced in* 1833.

Vol. II. Article IV. The latitude of the Cambridge Observatory, in Massachusetts, determined from transits of stars over the prime vertical observed during the months of December, 1844, and January, 1845, by William C. Bond, A. A. S., Major James D. Graham, A. A. S., and George P. Bond; by BENJAMIN PEIRCE, A. A. S.

Vol. III. Article VI. Some Methods of computing the Ratio of the Distances of a Comet from the Earth; by GEORGE P. BOND, Assistant at the Cambridge Observatory.

Vol. IV. Article VII. On some Applications of the Method of Mechanical Quadratures; by GEORGE P. BOND.

Vol. V. Article VII. On the Rings of Saturn; by GEORGE P. BOND.

Vol. VI. Part I. On the Use of Equivalent Factors in the Method of Least Squares; by GEORGE P. BOND.

There are many other papers in these volumes of great value, but of a less decided mathematical character, which we do not include for want of space.

Obituary.

Decease of Professor WILLIAM CRANCH BOND, *the distinguished Director of the Observatory of Harvard College, aged* 69 *years.*

THIS melancholy event occurred on the afternoon of Saturday, January 29, at the Observatory. An affection of the heart, with which he had been afflicted for several years, finally terminated in his sudden, but not unexpected, death. Thus has passed away one of the most eminent and successful cultivators of Astronomical Science. His various papers, published in the "Memoirs of the American Academy of Arts and Sciences," the various "Astronomical Journals," the "Annals of the Astronomical Observatory of Harvard College," his many brilliant discoveries in Astronomy, the invention of the "Spring Governor" for recording astronomical observations by means of electro-magnetism, are among the monuments of the ability, skill, and industry which have marked his honorable career. And his claims to distinction have been recognized by his contemporaries. In 1842 Harvard College gave him the honorary degree of Master of Arts; he was a Member of the American Academy of Arts and Sciences, Member of the American Philosophical Society of Philadelphia, and of the National Institute at Washington, Associate of the Royal Astronomical Society of London, Corresponding Member of the Institute of France, the Philomathic Society of Paris, the Accademia de' Nuovi Lincei at Rome, the Society of Natural Sciences at Cherbourg, and also of other learned societies.

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